# Zero and Infinity in Mathematics: Ayoga Kevalī and Siddha in Jain Philosophy

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## Introduction

*There are few subjects which accompany the consciousness at all times irrespective of its level of awareness. Mathematics is one of them for without it no action is possible but ironically it still remains a subject of worldly affairs and not of the ethereal.*

Like truth mathematical concepts too are innate part of nature and exist objectively irrespective of humans' quest to seek for them. For example: seasons change after a certain period of time, leaves fall and then burst out at exactly the right time of spring, flowers bloom etc. Similarly, all numbers have always been there in their totality, and in partiality, without humans being aware of them. Also, numbers portray a certain amount of a given thing or situation which is always physical but mathematics is philosophical too and all the numbers, including zero and infinity, somehow have their abstract states as well.

"For Plato, the complicated realities around us are, in a sense, shadows of perfect mathematical objects that exist quite separately, in the abstract world of mathematics. In that world, shapes and other geometric objects are perfect – points are infinitely small, lines are perfectly straight, planes are perfectly flat and so on."[1] Visit a desert, you will see sand dunes with beautiful patterns; visit the sea bed, you will find the same patterns. Nature works in patterns, some patterns are easier to understand while some seem impossible to decode, but there is a pattern behind each occurrence of the universe for sure. Mathematics needs to be understood and studied to evaluate each of our life's happening from tiny to big.

We can contemplate that if there can be different types of zero; as zero is always considered as a Dumbo whereas in reality it is the superhero. Perspective changes the way we look at a particular number, and in this context zero can be considered both zero and hero. When we contemplate about infinity, we consider it as only one and we don't divide it further however in the Jain philosophical text Dhavalā there is mention of eleven types of infinity.[2] Hence we must keep our mind open to newer ways of approaching Mathematics.

In Jain philosophical text *Gommaṭṭasāra Jīvakāṇḍa*, authored by Ācārya Nemicandra Siddhāntacakravartī, two types of Mathematics have been mentioned: "*laukika*, which is used to measure the physical and smaller aspects of the substance and *alaukika*, which is adopted to measure the micro and infinite dimensions of the substance".[3] Still we can only grasp a tiny part of it as like truth even Maths is a subject to be felt and even here the numbers work as mere signposts.

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We see many passionate teachers who teach Mathematics, a subject only a few are comfortable with, with different, easier and more creative approaches. This way, we can say that we all are equipped with it; we only need to find our very own unique way to understand this beautiful and philosophical subject. It is the most important subject of our lives, which provide a whole array of researches to decode the patterns hidden in the universe. Just like the zero and infinity can be compared with the states of the *ayoga kevalī* and *siddha* respectively.

**Importance of Mathematics**

*Notice your surroundings, the birds, the animals, your pets; and you will see they all calculate. Despite being unaware of numbers created by humans, they are somehow acquainted with calculations for even in taking a flight or jumping from one branch to another or attacking a prey or simply getting up from one place to sit on another, all require some sort of calculation.*

This is how important mathematics is. Without it nothing is possible, one need not to be a genius in the subject however everyone is inevitably acquainted with it; for without it no thought, no idea, no creation (be it art or otherwise) can take place. Not only that, "all things known have number – for without this, nothing could be thought of or known".[4] We think, we act under the influence and with the help of numbers. Numbers rule the physical realm and without a doubt the universe is the best in dealing with numbers and hence, the best mathematician, who keeps track of each of the tiny and mega happenings, acts and reacts accordingly and fairly.

Italian astronomer, physicist and engineer Galileo Galilei (1564-1642)[5] expressed, "the laws of nature are written by the hand of God in the language of Mathematics"[6]. Nature enacts in beautiful patterns and so does Mathematics, one cannot understand the workings of the universe without understanding Mathematics.

Whether the universe has a beginning or not, it is a separate topic of discussion but if it has a point of being created, it could have been due to numbers for sure. "In numbers lay power, even possibly the power that had created the universe. Numbers were the key to vast knowledge – the sort of knowledge that would raise one's soul to a higher level of immortality, where it would re-join the divine."[7] For numbers take one to a journey of possibilities in positive as well as negative directions, where there is infinity in both the directions. Further contemplation about numbers gives a hint that through a deeper understanding of them one can understand even the journey of the soul.

However just like rationality, numbers and hence Mathematics are the subjects of the worldly matters. Albert Einstein once said, "The intuitive mind is a sacred gift and the rational mind is a faithful servant. We have created a society that honours the servant and has forgotten the gift."[8] Reason is one of the synonyms for rationality and "reason could also be translated as calculation".[9] Mahāvīrācārya mentions, in his work *Gaṇita-Sāra-Saṁgraha,* that calculation

holds a necessary and respectable place in perceiving all the worldly matters, be it the technique of love-making, economics, understanding the musical notes or in theatre. Be it making food, constructing and designing buildings, writing a piece of poetic expression, understanding rationality, or any form of art. In all those transactions which relate to worldly, Vedic or (other) similarly religious affairs, calculation is of use.[10] "In the science of love, in the science of wealth, in music and in the drama, in the art of cooking, and similarly in medicine and in things like the knowledge of architecture."[11] It is well established too, that "the first natural law ever formulated mathematically was the relationship between musical pitch and the length of a vibrating harp string, and that it was formulated by the earliest Pythagoreans"[12]. Even in dance one requires tremendous calculative skills to catch every beat and perform accordingly, only a genius at calculation can be a magnificent dancer.[13] In the translation of *Gaṇita-Sāra-Saṅgraha*, Raṅgācārya says, "In prosody, in poetics and poetry, in logic and grammar and such other things, and in relation to all that constitutes the peculiar value of (all) the (various) arts: the science of computation is held high esteem."[14] The famous German writer Johann Wolfgang von Goethe (August 1749-March 1832) said, "Music is liquid architecture; Architecture is frozen music."[15] And as music is Mathematics, architecture is Mathematics too, only the form is different.

This way, we see that not only a person who is good at calculations using formulas can be considered genius in Mathematics, but also someone who is an artist of any form like dancing, painting, writing – poetry and prose, even a sportsperson, an architect, a chef, or one from any other profession can be regarded as a mathematician. This thought, this idea lifts a veil of ignorance from our consciousness and the vision to see the world, to understand mathematics in a new light becomes clearer. For nothing is possible without calculation.

**Zero – the real superhero**

*In the infinite ocean of numbers zero is the centre, one may either call it completely devoid of entity or having the true, absolute essence. The latter seems closer to define the power of zero.*

Irrespective of how zero is used in maths, pondering over it may lead one to a peaceful state, where there is no botheration of numbers, possessions, and baggage of physical as well as mental objects. Every positive along with negative integer has its roots in zero, irrespective of how far it is located from it. In the context of consciousness, we may say no matter how much one is lost or wandered; one is never really away from one's own, true nature, which is closer to a state similar to zero. Without zero the calculation of any value is not possible. In fact, value of every integer can be doubted as an absolute one but perhaps zero is the only one which has the absolute value of nothingness. One can even meditate upon zero to be like it.

Contrary to the popular belief about the discovery of zero "in Indian mathematics, the oldest known text to refer to zero is a Jain cosmological text entitled the *Lokavibhāga* ('The Parts of the Universe'), originally composed in Prakrit in fifth century CE."[16]

We misunderstand zero totally when we use it as a barb for someone who is incompetent. Zero is quite philosophical and rather it should be used as a compliment. Zero is the exact centre of the infinite ocean of numbers, where on one hand there are positive integers, on the other all negative. And perhaps zero is the only absolute, so much so that it is just a tiny step away from infinity. "Just as emptiness of space is a necessary condition for the appearance of any object, the number zero being no number at all is the condition for the existence of all numbers."[17] The moment one understands the gist of it, the whole concept of numerals starts to have a different meaning. As if they have found a base, a foundation to relate to.

"Zero is not a 'natural' candidate for acceptance as a number unlike other numerals. It requires a great leap of thoughts from the concrete to abstract."[18] Zero must have gone through a series of struggles to be accepted as an important part of the number family and without a doubt the family must have thought later that we were useless and incomplete without it. For the value of zero cannot be physical or understood with the rationale. It is something to be contemplate upon, to be felt through one's conscience.

"The Sanskrit word for zero, *śūnya*, means "void or empty". It describes a sense of 'absence' with the potential for growth."[19] However, zero often amuses us and its value adds on and takes away immensely in and from numbers when kept at different places; for example – it holds a different meaning when placed between two numbers, whereas it lessens the value of a particular number if placed before it, the more there are zeros after any given number, the higher it becomes in ranking. Also when a number is being multiplied by zero, zero is powerful enough to convert the number into zero no matter how higher the number is. Zero is beautiful to understand and seems to be a valid candidate to be look upon as a role model.

"*Śūnyatā* is the abstract noun of *śūnya* describing emptiness, void, etc. The practice of *śūnyatā* is recommended in projects involving writing of poetry, music composition, painting or any activity that springs from the mind of the artist."[20] For in the state of nothingness, in being present in the present moment, in the now, an artist becomes a creator. When the consciousness frees itself from the grip of mind and basks into the glory of itself, creation happens. This is how beautiful zero is; for sure there is a physical state which can be compared with the glory of it.

**Ayoga Kevalī- why Zero and Not Śūnya?**

*Don't get stuck on the level of words. A word is no more than a means to an end. It's an abstraction.*[21] - Echkart Tolle

The modern definition of zero is "a value of an independent variable that makes a function equal to zero. The absence of a measurable quantity. Having no magnitude or quantity."[22] However, zero is much beyond this definition; a philosophical approach is required to understand the magnitude of zero. Jainendra Siddhānta Kośa describes the nearest to zero state as *śunya* which is – "absence of all substances"[23]. And we know that in Jain philosophy substances are six in number; namely – *jīva* (the consciousness), *ajīva* (the non-living

matter), *dharma* (medium of motion), *adharma* (medium of rest), *ākāśa* (space) and *kāla* (time). According to this definition, *śunya* depicts only the *alokākāśa* (supra-cosmic space), whereas all the substances exist together in the *lokākāśa* (cosmic-space).

Hence, we see that there is a difference in the definitions of zero and *śūnya*, zero is closer to the state of the *ayoga kevalī*, whereas *śūnya* is absence of all substances. Being the purest state of consciousness, the state of the *ayoga kevalī* is nothing but consciousness, which depicts that there is only one substance left and even this substance cannot travel unto the top most part of the universe without the help of another substance which is *dharma dravya* (medium of motion).[24] Due to this definition zero becomes a close candidate to be compared with the state that can describe the glory of it and that is *ayoga kevalī.*

**The state of the Ayoga Kevalī**

Let us ponder over to find a physical state closer to zero and we will find the exact one in the last and the fourteenth step of the fourteen stages of spiritual development mentioned in Jain philosophy as *ayoga kevalī.* Not only that but also, we will see that it is soon going to attain the state of infinity which is described in the tenth chapter of the ancient text Tattvārtha Sūtra as *siddha*.

The journey to become zero is not an easy one, for one has to understand the whole karma theory, how it works and the possibility of attaining a state where there is absolutely no karma left. The consciousness and the karma are together since time immemorial, they are intertwined in a way that it is difficult to differentiate between the two and it is even more difficult for the consciousness to get rid of the same. Though the inflow (*āsrava*) and shedding (*nirjarā*) of karma keep occurring at all times, according to the being's thoughts, words and actions. However, at a higher spiritual level the consciousness becomes able to stop the inpour completely; this phenomenon is termed as *saṁvara* (inhibition)[25]. Tattvārtha Sūtra[26] says, "Omniscience arises when deluding karma is eliminated and, as a result, knowledge-covering, intuition-covering and obstructive karma are eliminated…. There is no fresh bondage because the causes of bondage have been eliminated and all destructive karmas have worn off."[27] This is a state where the series of cause and effect breaks and as a result cause as well as effect become nil which results into elimination of the *ghātī* karma (destructive karma) and only the *aghātī* karma (non-destructive karma) left, which shed only after completing their time of bondage with the *jīva* (being). This is the thirteenth and the second last step of the spiritual ladder and is called as *sayoga-kevalī* which means "omniscient with physical activity".[28]

When the soul becomes as motionless as a mountain rock, this state of soul is called *ayoga-kevalī.* "This state of absolute motionlessness is the fourteenth and the last stage of spiritual development in the instant before death. The soul then frees itself of the sensation, body, lifespan and status karmas and attains disembodied eternal liberation".[29] This is a state where the soul is at the exact zero and this stage lasts for a very few times only and then the consciousness departs on a journey to infinity.

According to Jain philosophy after complete annihilation of karma an individual becomes god. That state of god has two aspects – one is the liberated being with the body and the other is liberated being without the body. The first stage is called *arihanta* and the latter is called as *siddha*.[30] *Arihanta* (omniscient) is the state of *sayoga-kevalī* and *siddha* is the stage after *ayoga-kevalī*. One may have a query that why the state of *arihanta* is not compared with zero, it is because despite being *arihanta* (the supreme manifestation of being human), it has 85 *karma prakṛtis* left to be destroyed hence it is called *sayoga-kevalī*, and the moment it sheds these *karma prakṛtis*, it attains the state of the *siddha*. However, in text Dhavalā it has been mentioned that, "*sayoga-kevalī* and *ayoga-kevalī* both can be called *arihanta* as both have destroyed the destructible karma (*ghātīya karma*) and have observed the substances in their absolute completeness through omniscience"[31].

**Infinity in Jain Philosophy**

*One is that superficial that one cannot understand even the tiniest of infinity, however one feels it when one is immersed deep within.*

The omniscient perceives and knows everything that is there to know in the universe, in its true essence. Essence which has innumerable viewpoints, ways to perceive that particular thing's history, present and future; which results into humongous calculations; for the omniscient sees all at once. "The way number of matters, substances and even expressions have been explained strikingly through the notion of the omniscient, it has not been seen anywhere else. These calculations go beyond the computations hence they are represented by innumerable and infinite."[32]

Further understanding of innumerable and infinite clears that these are not subjects to be understood by the human rationale. "Number of subjects directly known by the omniscient are called innumerable and infinite."[33] However we can have a glimpse of them through gaining appropriate knowledge.

Contemplation about infinity may lead one towards questions like – what is infinity exactly? Does it have any direction? Are there types of infinity? Is it an abstract concept or it may have some physical reality? How can one relate to it? Does it exist in numbers only or in other concepts like thoughts, space, size also? Surprisingly one finds solutions to all these queries in the ancient texts of Jain philosophy. Jain thinkers have dived quite deeper into the philosophy of innumerable and infinite, for example – in Tattvārtha Sūtra[34] Ācārya Umāsvāti writes, "There are innumerable soul units in a soul"[35]. It is highly contemplative that the soul, despite being an abstract entity has innumerable soul units. This can be understood through an example which can also be the answer of the long seeked question about exact place of the soul in a being's body? According to Jain philosophy just like the oil pervades in the whole seed (and not in a specific part of the seed), the soul pervades in the whole (physical) body of a being.

Coming to the next query about infinity in space, Ācārya Umāsvāti[36] mentions: "There are an infinite number of space units in space"[37]. Here the important thing to notice about infinity is that space consists of space units, whereas soul consists of soul units; hence, the cardinal units are different for both. But at the same time, just like any other existent of the universe, these units share the same phases of origination, cessation and continuity.

According to Jain philosophy, "The number of units in clusters of matter may be numerable, innumerable or infinite."[38] Now, this is an extremely interesting fact about matter that it may vary from a single, smallest, indivisible unit of matter, which is known as *paramāṇu*, to possessing numerable, innumerable and infinite units of the same; which eventually decides the size of that particular thing.

We must also try to discriminate between innumerable and infinite before understanding their respective meanings; according to Jain text Dhavalā we can know their difference by this definition – "the number which finishes off if we keep deducting one number at once, one by one, is innumerable and the number, which does not finish by the same process, is infinite".[39] Not only that, but also infinity has been further subdivided into two parts in the mathematical text Gommaṭasāra Jīvakāṇḍa – "*sākṣaya ananta*, the infinite quantity that comes to an end when kept spending and *akṣaya ananta*, the infinite quantity that irrespective of not adding any new amount into it, does not end even after perpetual expenditure"[40]. Not only that Jain thinkers even mention that infinity can be of three types: "low-grade, self-raised and infinite-infinite; each of these three can be further subdivided into three parts as minimum, intermediate and maximum"[41]. Understanding infinity is indeed tougher than it seems, for the quantity we assume as the ultimate one, beyond which nothing exists, also has subdivisions.

The thinkers don't stop here only they divide infinity further, into eleven more parts, considering its uses. In Dhavalā these eleven variants are:[42]

i. **Nāma ananta**: to use infinity as a word, without knowing if a particular object is infinite or not. For example: expressing one's love for someone as infinite.
ii. **Sthāpanā ananta**: to implant or establish infinity in an object however that is not the real infinity.
iii. **Dravya ananta**: this has been divided into two parts as *āgama dravyānanta* and *noāgama dravyānanta*. They are being described as: one who knows the texts that contains the knowledge of infinity but who is unaware of its usage at present and one who will gain knowledge of infinity in future respectively.
iv. **Śāśvata ananta**: who/which is eternal since time immemorial. For example: the six substances of the universe.
v. **Gaṇanā ananta**: number which cannot be counted.
vi. **Apradeśa ananta**: the smallest particle of matter known as *paramāṇu* for it only occupies single unit of space.
vii. **Eka ananta**: this is the infinity which can be seen only in a straight line and in one direction only.
viii. **Ubhaya ananta**: this is the infinity which can be observed in two directions but in a straight line only.

ix. **Vistāra ananta**: that infinity which has expansion in all directions. For example: the space.
x. **Sarva ananta**: seeing the space as a form of wealth will lead one to this kind of infinity.
xi. **Bhāva ananta**: one who knows the texts that contains the knowledge of infinity and is absolutely aware of that too.

This way, we see that infinity is described in incredible ways in Jain texts. Its study initiates a newer, deeper thought process in one's brain. Where one is bound to contemplate over it again, with answers like – it has types, it can be physical as well as abstract, even infinity can be minimum, moderate and maximum, it can expand either in one or two directions or in all directions, it can be present not only in numbers but also in thoughts etc.

**Siddha: Souls that have attained infinity**

*O what a state, where pervades nothing but purest of consciousness*
*Infinity of knowledge, perception, bliss and power*
*Mere imagination fills one with inner happiness*

On a serious note if we ponder over a state similar to this, irrespective of if it exists or not, mere imagination fills us with peace, serenity and lightness. A state, to attain which, is the ultimate aim of a *jīva's* journey of lives, deaths and rebirths. A halt, a stop where there is nothing beyond, no questions, no looking further but only the real, purest essence. Hence, let us embark on a journey to know godhood in Jain philosophy, a state of infinity. Prior to thinking of physicality of infinity, one must think if there is a place in the cosmos which has the ability to accommodate those who have attained so. In Jain philosophy, one finds that place at the tip of the universe. *Siddha* are those – "who have destroyed the restriction of all eight types of karma, are located at the top most realm of the universe, are absolute and eternal"[43]. Another beauty of Jain philosophy is that it erases the discriminatory barriers of cast and creed when it mentions very clearly that each soul is worthy to attain the state of a *siddha*, the state of godhood as there is not only one or few *siddha*, but they too are infinite in number. As many beings have attained salvation since time immemorial and many will continue to achieve this state following the path of right knowledge, perception and ethics. It is simple yet profound that whosoever pursue this path will be liberated, however that is another matter of discussion that rare are those who are keen to do so or are even aware of this path.

According to Jain philosophy[44], "The elimination of all types of karma is liberation"[45]. Also known as *mokṣa*, salvation and emancipation. The path to liberation is simple, no karma leads to it. When the *jīva* is in the right direction, it works inevitably towards it. A gradual process of gaining knowledge, clarification of perception and ethical enhancement occurs, which in turn results into shedding of (destructible) karma. The *jīva* keeps moving towards its true nature of bliss, until it achieves its infinity and then it stays in that state forever. In Tattvārtha Sūtra,[46] Ācārya Umāsvāti mentions, "When all the karmic bondage is eliminated, the soul

soars upwards to the border of cosmic space"[47]. Just like a castor seed, such is the nature of liberation.

To understand the place where the *siddhas* reside forever in their true nature one may study the cosmology (of the universe) described in Jainism. The definition of *loka* or the universe is: "that finite, beginning less and inartificial part of space, which is surrounded, at all sides, by the infinite space, in which all the six substances (*jīva*, *pudgala*, *dharmāstikāya*, *adharmāstikāya*, *ākāśa* and *kāla*) can be seen, is *loka*"[48]. To locate the exact residence of the liberated beings "there is *siddhaloka* at the crown of the *loka* where liberated beings are situated with their body which is nothing but knowledge"[49].

**Conclusion**

*O mathematics, one is obliged to you for being there to help one decoding the secrets of the universe*
*Only because of you, from the countless cycles of lives, one can now traverse*

Just as zero can be the symbol to represent the state of *ayoga-kevalī* and infinity can be used for the state of the *siddha*. Each mathematical symbol, formula, equation may be used to represent complicated human behaviour, mental illnesses and disorders, along with exploring the different dimensions of the universe and if there are not enough formulas to represent so, new formulas can be discovered to deal with the same. In short, there are innumerable chances for innovation in mathematics.

For example, profound study of infinity may boggle minds of even the great thinkers. It raises questions like, "how could any number 'equal' infinity? Infinity was no place you could reach, no quantity you could plug into an equation; there was no 'last number'. So to understand how a mathematical expression behaved 'at' infinity was to explore an elusive and mysterious terrain out beyond all seeing."[50] And as we saw the subject of infinity has been dealt in quite detail in Jain philosophical texts.

For the Indian genius Srinivasa Ramanujan, "numbers and their mathematical relationships fairly threw off clues to how the universe fit together. Each new theorem was one more piece of the Infinite unfathomed"[51]. With this thought from this genius in mind, we can definitely embark on a journey of explorations in the field of research in maths. Through his first love of 'infinite series' Ramanujan provided a gateway, for us, to peep into infinity. Imagine "square roots of square roots of square roots of… and fractions of fractions of fractions of…"[52] wouldn't they take us to infinity of the micro world? Or we can have a look at the same phenomenon the other way around, 'multiples of multiples of multiples of… and additions of additions of additions of…' to enter into the macrocosm. To conclude mathematics can decode the truth, the absolute truth.

**Acknowledgments**

i. The Facebook page 'Jain 108 Mathemagics'
ii. Documentary 'Nature's Mathematics' presented by Prof. John Adam, Old Dominion University
iii. Dr. Snehrani Jain
iv. Dr. Yashwant Jain, more information about him @ yashkjain.blogspot.com
v. Dr. Shugan Chand Jain and Dr. Shrinetra Pandey for helping in improvising the paper by their valuable feedback

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*vyāpārastatra sarvatra saṁkhyānamupayujyate//*
*kāmatantre'rthaśāstre ca gāndharve nāṭake'pi vā/*
*sūpaśāstre tathā vaidye vāstuvidyādivastuṣu//*" (Gaṇita-Sāra-Saṁgraha, Saṁjñādhikāra 9-10)
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*kalāguṇeṣu sarveṣu prastutaṁ gaṇitaṁ paramam//*" (Gaṇita-Sāra-Saṁgraha, Saṁjñādhikāra 11)
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[25] "*āsravanirodhaḥ saṁvaraḥ*", Ibid 9/1.
[26] "*mohakṣayāj-jñāna-darśanāvaranāntarāyakṣayācca kevalam/ bandhahetvabhāva-nirjarābhyām/*", Tattvārtha Sūtra 10/1-2.
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[34] "*Asaṁkhyeyāḥ pradeśā dharmādharmayoḥ/ jīvasya/",* Tattvārtha Sūtra 5/7-8.
[35] Tatia, Nathmal. Op. Cit. 2011: 125.
[36] "*Ākāśasyānantāḥ",* Tattvārtha Sūtra 5/9.
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[38] "*Saṁkhyeyā'saṁkhyeyāśca pudgalānām",* Tattvārtha Sūtra 5/10.; Tatia, Nathmal. Op. Cit. 2011: 126.
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